AIAA 80-0792R

# Wing/Store Flutter with Nonlinear Pylon Stiffness

Robert N. Desmarais* and Wilmer H. Reed III†
*NASA Langley Research Center, Hampton, Va.*

**Recent wind tunnel tests and analytical studies show that a store mounted on a pylon with low pitch stiffness provides substantial increase in flutter speed of fighter aircraft and reduces dependency of flutter on mass and inertia of the store. This concept, termed the decoupler pylon, utilizes a low-frequency control system to maintain pitch alignment of the store during maneuvers and changing flight conditions. Under rapidly changing transient loads, however, the alignment control system may allow the store to momentarily bottom against a relatively stiff backup structure in which case the pylon stiffness acts as a hardening nonlinear spring. Such structural nonlinearities are known to affect not only the flutter speed but also the basic behavior of the instability. This paper examines the influence of pylon stiffness nonlinearities on the flutter characteristics of wing-mounted external stores.**

## Nomenclature

| | |
|---|---|
| $K$ | = spring constant of linear soft spring |
| $K_e$ | = equivalent linear spring constant of nonlinear spring |
| $M$ | = elastic restoring moment about store pitch axis |
| $M_1$ | = fundamental Fourier component of store pitch moment |
| $M_0$ | = static moment required to deflect store against hard spring |
| $\bar{M}$ | = static preload moment |
| $N$ | = ratio of high stiffness spring constant to low stiffness spring constant |
| $t$ | = time |
| $\bar{t}$ | = time at which store contacts hard spring |
| $V$ | = flutter speed |
| $V_{nom}$ | = flutter speed of linear system with nominal-design, stiff pylon (spring constant $= NK$) |
| $\delta$ | = describing function ($\delta = K_e/K$) |
| $\delta^*$ | = describing function when $\bar{M}/M_0 = 1.0$ |
| $\theta$ | = store pitch angle |
| $\theta_1$ | = amplitude of sinusoidal store pitch oscillation |
| $\theta_0$ | = pitch angle at which store contacts hard spring |
| $\bar{\theta}$ | = static store pitch deflection due to preload |
| $\omega$ | = circular frequency |

## Introduction

ANALYTICAL investigations of aeroelastic systems are usually based on linear theory which assumes both the structural and aerodynamic properties to be independent of the amplitudes of oscillation. Aircraft structures typically exhibit nonlinearities, however, such as backlash or kinematic deflection limits in moving control surfaces and in the connecting structure between wing and external stores. Studies of flutter of wings with control surfaces that contain structural nonlinearities (Refs. 1-5) have shown that nonlinearities can affect not only the flutter speed of the system but also the characteristics of flutter motion. Similar studies in Ref. 6 investigate the effects of control system nonlinearities, such as actuator force or deflection limits, on performance of an active flutter suppression system. Whereas flutter of a linear system is characterized by an exponential growth of oscillation amplitude with time, flutter of a nonlinear system may be amplitude limited. On the other hand, a nonlinear system which is stable with respect to small disturbances may be unstable with respect to large ones. Interest in this particular problem stems from studies in Ref. 7 of a passive wing/store flutter suppression concept known as the "decoupler pylon." These studies and others have shown that a store mounted on a pylon with low pitch stiffness can provide substantial increase in flutter speed and reduce the dependency of flutter on the mass and inertia of stores relative to that of stiff-mounted stores. By decoupling the influence of store pitch inertia on wing torsion modes, the frequency separation between flutter-critical modes is increased and the flutter speed is increased as indicated in Fig. 1.

The decoupler pylon uses a low frequency control system to compensate for changes in static deflections of the store that would otherwise exist during maneuvers or airspeed changes. Depending on the time constant of the store alignment system and the rate of change of load, however, the store may deflect enough to exceed the linear range of the soft pitch spring and "bottom" against a relatively stiff backup structure. Under such conditions, pylon pitch stiffness varies in a nonlinear manner which depends on both the static preload and oscillation amplitude of the store.

## Nonlinear System

An idealized representation of the nonlinear pylon suspension system is shown in the upper part of Fig. 2. The store is suspended from the wing by a pivot located near the wing elastic axis. The store pitch frequency is controlled by a soft linear spring. The spring stiffness $K$ is chosen to make the uncoupled store pitch frequency somewhat lower than the fundamental wing bending frequency when the store is rigidly mounted (see Ref. 7). A static preload $\bar{M}$ is assumed to act on the store as a result of loads such as a high-g pitch-up maneuver. When the preload exceeds a value $M_0$, the associated static pitch displacement causes the pylon to contact mechanical stops at $\theta = \theta_0$. The stiffness then increases by a factor $N$ over that of the soft spring. This situation results in the load displacement curve shown in the lower part of Fig. 2.

In this paper the preload and resulting displacement are such that the discontinuity in the load displacement curve at negative $\theta$ values is never reached so only the positive discontinuity, $\theta_0$, is analyzed. The nonlinear effects of damping are ignored because studies in Ref. 7 showed that flutter is insensitive to pylon damping.

## Describing Function Analysis Technique

The analysis technique used is the "describing function" or equivalent linearization method (see Refs. 2 and 4). In par-

Presented as Paper 80-0792 at the AIAA/ASME/ASCE/AHS 21st Structures, Structural Dynamics and Materials Conference, Seattle, Wash., May 12-14, 1980; submitted July 8, 1980; revision received May 1, 1981. 

*Aero-Space Engineer, Loads and Aeroelasticity Division. Member AIAA.

†Chief Scientist, Loads and Aeroelasticity Division. Associate Fellow AIAA.

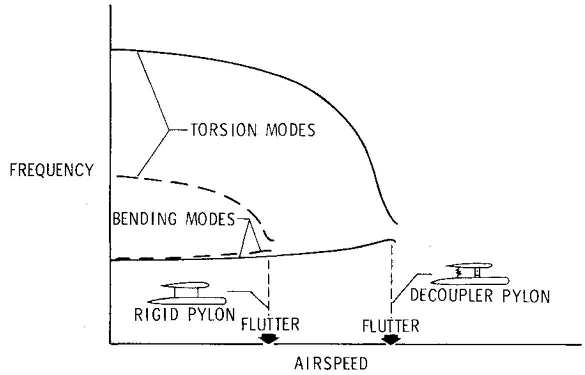


**Fig. 1 Effect of decoupler pylon on flutter speed.**

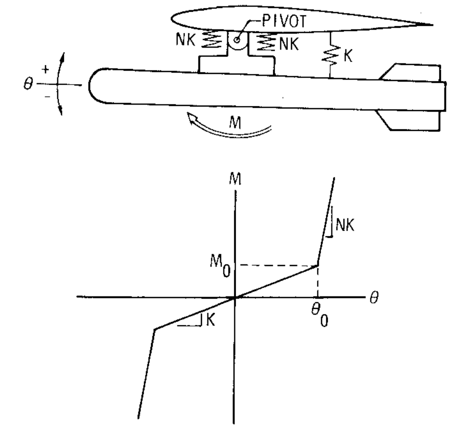


**Fig. 2 Decoupler pylon with nonlinear pitch stiffness.**

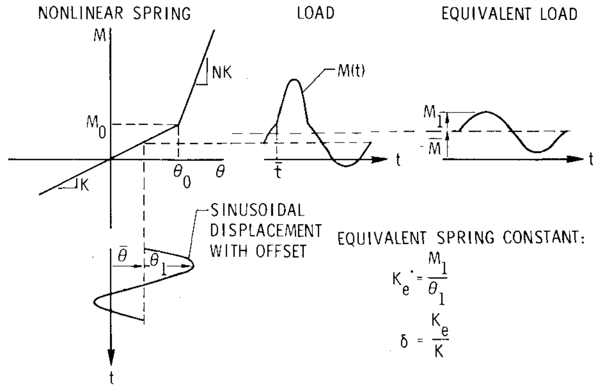


**Fig. 3 Representation of nonlinear spring by an equivalent linear spring.**

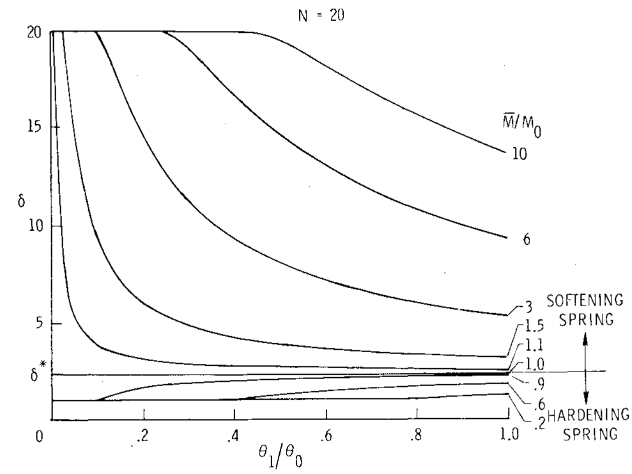


**Fig. 4 Describing function for bilinear spring ($N = 20$).**

ticular, Ref. 4 presents comparisons of the results of the describing function technique with results using a complete nonlinear solution of the same problem to illustrate the validity of the method.

The basis of the describing function method is to assume a sinusoidal displacement and then compute the load developed in the nonlinear spring. The spring load is then expanded into a Fourier series. The fundamental component of the series is retained, and higher harmonics, which are assumed to be negligible, are discarded. The spring constant of the equivalent linear spring is then determined by obtaining the ratio of the coefficient of the fundamental component of load to the displacement amplitude, $K_e = M_1/\theta_1$, where

$$\theta = \bar{\theta} + \theta_1 \sin\omega t$$

$$M = \bar{M} + \sum_{n=1}^{\infty} M_n \sin n\omega t$$

In this type of analysis, for each preload and displacement amplitude the problem is linear, consequently a linear flutter analysis may be made. The nonlinearity only appears as a change in equivalent linear spring constant when the preload or displacement amplitude is changed.

With reference to Fig. 3, the describing function is computed as follows: The load $M$, developed in the nonlinear spring, is expressed in terms of the store oscillation amplitude $\theta_1$ and the time $\bar{t}$ at which the store contacts the hard spring. This load is integrated over one cycle to give an expression for the preload, $\bar{M}$ (the zeroth Fourier coefficient). For each specified preload $\bar{M}$, this expression is then solved for $\bar{t}$. This information is then used to compute the first few Fourier coefficients. All the Fourier cosine coefficients integrate to zero because the load function is symmetric about the zeros of the cosine function. The first Fourier sine coefficient, $M_1$, is used to compute the equivalent spring constant, $K_e = M_1/\theta_1$, and the describing function, $\delta = K_e/K$. The second Fourier sine coefficient, which has a very small magnitude, is used to assess the validity of the method. No other Fourier coefficients were computed.

Figure 4 is a plot of describing function $\delta = K_e/K$ vs amplitude for various preload moments. Note that when the preload is less than $M_0$ and the static plus dynamic deflection is less than $\theta_0$, the equivalent spring constant is the same as that of the linear soft spring, i.e., $\delta = 1.0$. With increasing oscillation amplitude, the system begins to contact the hard spring when $\theta = \theta_0$ and the equivalent spring then stiffens as the amplitude increases. This transition between a linear soft spring and a nonlinear hardening spring occurs in Fig. 4 at the oscillation amplitudes where the curves for constant preload break away from the $\delta = 1.0$ line. Conversely, when preload exceeds $M_0$, the equivalent spring constant is the same as the linear hard spring ($\delta = 20$) for small amplitude oscillations. However, as oscillation amplitudes increase and deflections enter the soft spring range, the stiffness is characterized by that of a nonlinear softening spring.

An interesting and significant feature of a bilinear spring is apparent in Fig. 4. When the preload exactly matches the "bottoming" load, $\bar{M}/M_0 = 1.0$, the equivalent spring constant is independent of oscillation amplitude. In other words, the system behaves as though the spring were linear for all oscillation amplitudes. The describing function corresponding to this transition region between a hardening spring and a softening spring is designated $\delta^*$, and its value depends on $N$, the ratio of the two spring constants. For the case shown in Fig. 4, $N = 20$ and $\delta^* = 2.412$. The variation of $\delta^*$ with $N$ is indicated in Fig. 5.

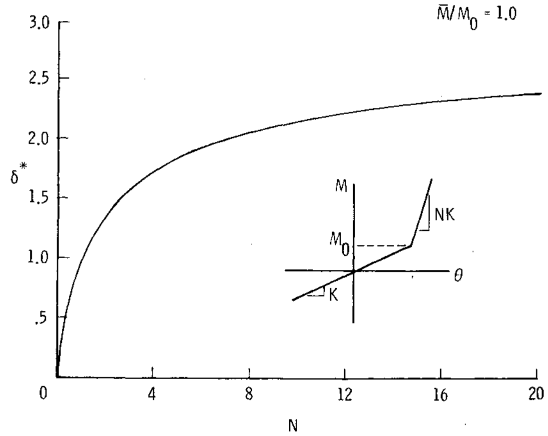


Fig. 5 Describing function for transition between hardening spring and softening spring that occurs when $\bar{M}=M_0$.

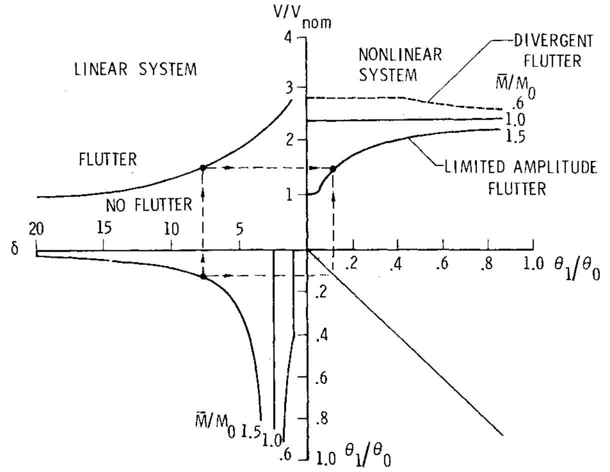


Fig. 6 Illustration of method for determining wing/store flutter boundaries with nonlinear pylon stiffness. $N=20$.

## Flutter Calculations

The describing function method is incorporated into flutter calculations as follows. First, the flutter velocity is computed as a function of equivalent pylon pitch stiffness using any standard linear flutter analysis technique. For convenience, this flutter boundary is expressed in terms of nondimensional ratios $V/V_{nom}$ and $\delta=K_e/K$ where $V_{nom}$ is the flutter speed of a linear system with nominal pylon pitch stiffness typical of current fighter design practice (in this case, 20 times that of the low pitch stiffness pylon). Then, the family of curves relating the describing function to amplitude and preload (Fig. 4), is cross plotted against the flutter boundary to eliminate $\delta$. The result is a family of flutter boundaries (velocity vs displacement amplitude curves), one for each preload. This procedure is illustrated graphically in Fig. 6. The linear theory flutter boundary is the upper left part of the figure and the describing function (Fig. 4) is plotted below it. Flutter boundaries for the nonlinear system are in the upper right part and the dashed lines with arrows indicate how they are related.

Several points can be made regarding the flutter boundaries shown in Fig. 6 for various preloads and oscillation amplitudes. Consider first the case where static deflections due to preload lies within the linear range of the decoupler pylon, i.e., $\bar{M}/M_0<1.0$. If the system is initially at rest, the flutter onset speed is the same as that for the linear system, i.e., approximately $V=2.8\ V_{nom}$. However, for speeds within the interval $2.3<V/V_{nom}<2.8$, the system can experience divergent flutter oscillations if a sufficiently large disturbance causes oscillations into the destabilizing stiff-spring range. For example, in Fig. 6 with a preload of $\bar{M}/M_0=0.6$, it can be seen that the degrading effects of stiffness nonlinearity on flutter begin to appear when the disturbance amplitude becomes greater than $\theta_1/\theta_0=0.4$. For $V>2.8\ V_{nom}$, divergent oscillations occur for any disturbance. The line separating initial disturbances which cause flutter from those which do not is shown dashed. Note also that when $\bar{M}/M_0=1.0$, the flutter velocity is independent of the magnitude of the disturbance, the same as for a linear system. This interesting feature is, of course, a consequence of the describing function being independent of amplitude at $\bar{M}/M_0=1.0$ for a bilinear spring, as was discussed earlier.

Consider next, the range of preloads which cause static deflections greater than the limits of the soft spring, i.e., $\bar{M}/M_0>1.0$. Flutter onset under these conditions occurs at $V_{nom}$, the flutter velocity of the linear system with stiff pylon. However, unlike a linear system, where oscillation amplitude grows indefinitely and exponentially with time, the flutter amplitude for this nonlinear system is self-limiting because as amplitude increases the resultant softening effect of the nonlinear spring (see Fig. 4) tends to stabilize the system. This can be illustrated, for example, by the flutter point shown in Fig. 6 where, for $\bar{M}/M_0=1.5$, the flutter amplitude is limited to about $0.1\ \theta_0$.

By physical reasoning it can be deduced that the sign of the slope of the velocity vs deflection amplitude curves, $dV/d\theta_1$, determines whether flutter oscillations will be divergent or of limited amplitude: a curve with positive slope indicates limited amplitude flutter; a curve with negative slope indicates the disturbance amplitude that must be exceeded to cause divergent flutter.

## Applications

In this section of the paper some analytically predicted effects of pylon stiffness nonlinearities on wing/store flutter are presented for two configurations: the F-16 and a flutter research model. Wind tunnel tests of both configurations with linear decoupler pylons have been conducted in the Langley Transonic Dynamics Tunnel.

### F-16 Flutter Model

The F-16 store configuration considered in this example is designated configuration 32 in Ref. 6. It consists of a GBU-8B store carried at wing station 120, and an AIM-9 missile at each wing tip. The GBU-8B stores are mounted on decoupler pylons which give an uncoupled store pitch frequency of 4.0 Hz (on the full-scale airplane). This frequency, selected on the basis of a criterion of Ref. 7, is approximately 70% of the first antisymmetric wing bending frequency with the store rigidly attached. The pitch stiffness when the system bottoms against mechanical stops is taken to be the stiffness of the nominal F-16 pylon design, which is 20 ($N=20$) times greater than that assumed for the decoupler pylon. A linear flutter analysis for F-16 configuration 32 was performed by General Dynamics, Fort Worth, Tex., with varying pylon pitch stiffnesses. Results of the analysis for antisymmetrical flutter (the most critical mode) at Mach number 0.9 are presented on the left side of Fig. 7. The companion curves plotted on the right side of Fig. 7 is a family of flutter boundaries which account for pylon stiffness nonlinearities. These flutter boundaries are for the same configuration as those previously presented, in abbreviated form, in Fig. 6 to illustrate the analysis procedure. Thus, earlier comments on Fig. 6 regarding divergent flutter and limited amplitude flutter are applicable to Fig. 7 as well.

To put in better perspective the magnitude of static deflections a decoupler pylon with soft pitch stiffness might experience in pitch-up maneuvers, some calculations have been made for the F-16 configuration considered in Fig. 7. In these calculations the benefits of an alignment control system were neglected and the most critical combinations of design maneuvers specified in Mil Spec MILA-8591E (Ref. 8) were as-

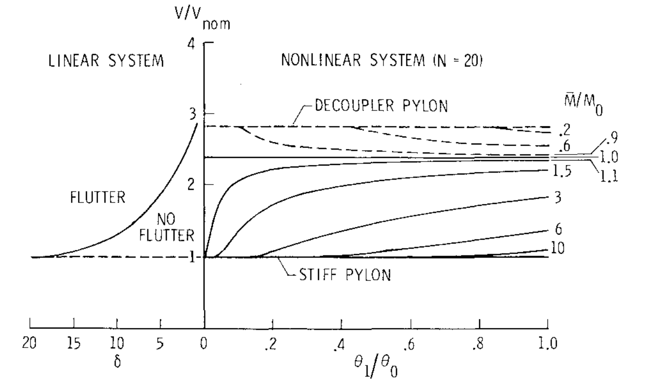


**Fig. 7 Calculated flutter boundaries for ¼-scale F-16 model with nonlinear pylon stiffness (Configuration 1, Ref. 6).**

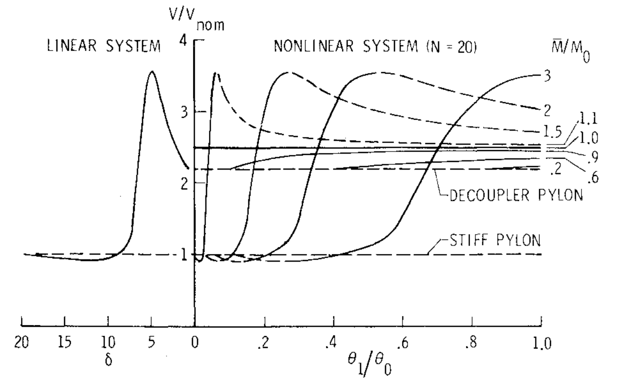


**Fig. 8 Calculated flutter boundaries for flutter research model with nonlinear stiffness (Configuration 32, Ref. 7).**

sumed, i.e., pitch acceleration, +4 rad/s; normal acceleration, +11.5 $g$'s and longitudinal offset of store center of gravity, 3.5 in. forward. The static pitch deflection of the GBU-8B store predicted for this extreme pitch-up maneuver was approximately 1.4 deg. Therefore, even though the decoupler pylon has a pitch stiffness that is low relative to the nominal pylon design stiffness, static deflection of the store during maneuvers is small. The bounds on store pitch deflection for other types of transient loads, such as gusts, are also being analyzed in a separate study. It appears, however, the main requirement for a store alignment control system is to compensate for the quasi-steady variation in drag loads due to changing flight conditions.

**Decoupler Pylon Research Model**

The second configuration analyzed is the decoupler pylon research model used in studies reported in Ref. 7. The model is a cantilevered rectangular wing with a store mounted at the 80% semispan station. The linear-system flutter (velocity) boundary for the model as a function of pylon pitch stiffness shown in Fig. 8 was derived from Fig. 6 of Ref. 7. As in the previous example, pylon stiffness is expressed in terms of the describing function for a bilinear spring with $N = 20$.

Note that in contrast with the F-16 flutter boundary (Figs. 6 and 7) which decreases monotonically with increasing pylon pitch stiffness, the linear-system flutter boundary in Fig. 8 is characterized by a peak near $\delta = 5.0$. This peak falls in the region of pylon stiffness where there is frequency coincidence between the uncoupled pylon pitch mode and the wing fundamental bending mode. Because of this peak in the linear-system flutter boundary, the effects of stiffness nonlinearities on flutter are somewhat different from, and more complicated than, the case shown previously. Again solid lines are used to show the magnitude of limited-amplitude flutter and dashed lines to separate disturbance regions which led to either stable motion, or divergent flutter oscillations. For this nonlinear system, limited amplitude flutter can occur for any preload condition; in the previous example, it occurred only when $\bar{M}/M_0 < 1.0$.

## Conclusions

This paper investigates the effects of pylon stiffness nonlinearities on the flutter characteristics of wings with externally mounted stores. In particular, the focus of the paper is on a passive wing/store flutter suppression concept known as the decoupler pylon. This concept uses a soft pylon pitch spring to decouple the store pitch mode from wing torsion modes assisted by a low frequency active control system to reduce static deflections of the store due to maneuvers and changing flight conditions. The structural nonlinearity under consideration is associated with bottoming of the system against a relative stiff backup structure as a result of excessive static and/or dynamic deflections of the store in pitch. By use of an approximate analysis technique (describing function technique), the nonlinear flutter behavior of two wing configurations with external stores is studied. On the basis of these studies the following conclusions may be drawn:

1) If the store static pitch deflection due to preload falls within the linear stiffness range of the decoupler pylon ($\bar{M}/M_0 < 1.0$), the flutter speed is substantially greater than (more than twice) the flutter speed with a nominal stiff-pylon design. When store pitch oscillations are superimposed on this static deflection, causing the system to bottom against a stiff backup structure, the flutter speed is changed but remains well above the flutter speed of the nominal stiff pylon.

2) If the store static pitch deflection due to preload exceeds the linear range of the soft pitch spring ($\bar{M}/M_0 > 1.0$), flutter onset occurs at the same speed as for the linear system with stiff pylon; however, the flutter amplitude is limited owing to the stabilizing effect of the softening pitch spring with increasing deflection amplitude.

3) If the static preload exactly equals the load required to bottom the system ($\bar{M}/M_0 = 1$), the flutter speed becomes independent of amplitude, as in a linear system, but is a function of $N$, the ratio of the high stiffness spring constant to the low stiffness spring constant.

4) Some sample calculations for an F-16 fighter performing a design-limited pitch-up maneuver were made to determine the static pitch deflections of a decoupler-pylon mounted store. The maximum predicted deflection (without an alignment control system) was 1.4 deg. Thus the requirement for an active alignment control system to avoid excessive static deflection of the store appears to be governed more by drag loads than by maneuver loads.

## References

[1] Woolston, D.S., Runyan, H.L., and Andrews, R.E., "An Investigation of Effects of Certain Types of Structural Nonlinearities on Wing and Control Surface Flutter," *Journal of the Aeronautical Sciences*, Vol. 24, Jan. 1957, pp. 57-63.

[2] Shen, S.F., "An Approximate Analysis of Nonlinear Flutter Problems," *Journal of the Aero/Space Sciences*, Vol. 28, Jan. 1959, pp. 25-32, 45.

[3] Breitbach, E., "Effects of Structural Nonlinearities on Aircraft Vibration and Flutter," AGARD-R-665, Jan. 1978.

[4] Laurenson, R.M., and Trn, R.M., "Flutter Analysis of Missile Control Surfaces Containing Structural Nonlinearities," *AIAA Journal*, Vol. 18, Oct. 1980, pp. 1245-1251.

[5] Breitbach, E.J., "Flutter Analysis of an Airplane with Multiple Structural Nonlinearities in the Control System," NASA TP-1620, Jan. 1980.

[6] Peloubet, R.P. Jr., Haller, R.L., and Bolding, R.M., "F-16 Flutter Suppression System Investigation," *Proceedings of the AIAA/ASME/ASCE/AHS 21st Structures, Structural Dynamics & Materials Conference*, Seattle, Wash., May 1980, pp. 620-634.

[7] Reed, W.H. III, Foughner, J.T. Jr., and Runyan, H.L., "Decoupler Pylon: A Simple, Effective Wing/Store Suppressor," *Journal of Aircraft*, Vol. 17, March 1980, pp. 206-211.

[8] "Military Specification MIL-A-8591E Airborne Stores, Associated Suspension Lugs, and Aircraft-Store Interface (Carriage Phase); General Design Criteria," *ASME*, Jan. 1976.